UNIVERSITY OF TORONTO
Faculty of Arts and Science
JUNE 2015 EXAMINATIONS
ECO381H1S – Personnel Economics
Duration 2 hours
(Instructor: J. Kantarevic)

Instructions

The test consists of seven questions, each worth five points. Please show all your work. Non-programmable calculators are allowed. Good luck!

1. An employee produces output according to q=e+u, where e is employee's effort that cannot be observed by the employer and u is a random variable with a mean of zero and a variance of 1. The employee's cost of effort is given by $c(e)=0.5e^2$. The employee is risk averse, with the coefficient of absolute risk aversion equal to 1, while the employer is risk neutral. The outside option is 0 for both the employer and the employee. In addition to the output q, the employer may also obtain an additional signal of employee's performance, denoted by y, where y has a mean of zero, a variance of 1, and its covariance with q is equal to 0.5.

   a. (2 points) What is the employer's maximum expected profit (E[q-w]) if the employer uses only q to design an optimal incentive contract (i.e. w=a+bq)?
   b. (1 point) Should the employer use both q and y to design an optimal incentive contract?
   c. (2 points) If the employer uses both q and y to design an optimal incentive contract (i.e. w=a+bq+cy), what is the optimal value of c?

Answer

a. (2 points) The employer designs a contract (a,b) that will maximize her expected profit, E[q-w], subject to the participation constraint (PC), E[U]≥R, and the incentive compatibility constraint (ICC), e maximizes E[U] given the contract (a,b). The employee's expected utility is $E[U]=E[w]-RP^A-c(e)=a+be-0.5(1)b^2(1)-0.5e^2$. Therefore, the ICC becomes b=e and the PC becomes $E[w]=0.5b^2+0.5e^2$. The employer's expected profit is E[q-w]=e-E[w]. Substituting from the ICC and PC, this becomes $b-b^2$. Therefore, the optimal contract has b=0.5 and the expected profit is equal to 0.5-0.25=0.25.
b. (1 point) According to the informativeness principle, the employer should use y to design an optimal incentive contract because y is correlated with q.
c. (2 points) The employer now designs a contract (a,b,c) that will maximize her expected profit, E[q-w], subject to the participation constraint (PC), E[U]≥R, and the incentive compatibility constraint (ICC), e maximizes E[U] given the contract (a,b,c). The employee's expected utility is $E[U] = E[w] - RP^A - c(e) = a+be-0.5(1)(b^2(1)+c^2(1)+2bc(0.5))-0.5e^2$, or $E[U]=a+be-0.5b^2-0.5c^2-0.5bc-0.5e^2$. Therefore, the ICC becomes b=e and the PC becomes $E[w]= 0.5b^2+0.5c^2+0.5bc +0.5e^2$. The employer's expected profit is E[q-w]=e-E[w]. Substituting from the ICC and PC, this becomes $b- b^2-0.5c^2-0.5bc$. The first-order condition for b is then 1-2b-0.5c=0 and for c is –c-0.5b=0, which yields b≅0.6 and c≅-0.3.

2. The employee's contribution to the firm value is given by $q=q_1+q_2$, where q_1 represents the 'quantity' aspect of the job and q_2 represents the 'quality' aspect. The quantity and quality depend on the employee's effort according to $q_1=e_1+u_1$ and $q_2=e_2+u_2$, where e_1 and e_2 is the time allocated to each aspect of the job that cannot be observed by the employer, and u_1 and u_2 are two random variables, each with a mean of 0 and a variance of 1. The covariance of u_1 and u_2 is 0.5. The employee is risk-averse, with the coefficient of absolute risk aversion equal to one, while the employer is risk neutral. The employee's cost of effort is given by $0.5(e_1^2+ e_2^2)$. The outside option is 0 for both the employer and the employee. The compensation contract is of the form $w=a+b_1q_1+b_2q_2$.

a. (1 point) Write down the employee's expected utility, using the information given in the question.
b. (1 point) Using the incentive compatibility constraint, explain how each of e_1 and e_2 depend on the optimal contract (a,b_1,b_2)?
c. (1 point) Write down the employer's expected profit that incorporates both the participation and incentive compatibility constraints for the employee.
d. (1 point) What is the optimal value of b_1 and b_2?
e. (1 point) Given the optimal contract, what is the employee's expected contribution to the firm's value (E[q])?

Answer

a. (1 point) $E[U]=E[w]-RP^A-c(e_1,e_2)$. Now, $E[w]= a+b_1e_1+b_2e_2$. Further, $RP^A=0.5rVar[w]$ and $Var[w]=(1)b_1^2+ (1)b_2^2+2b_1b_2(0.5)$. Therefore, $E[U]= a+b_1e_1+b_2e_2-0.5b_1^2- 0.5b_2^2-0.5b_1b_2-0.5(e_1^2+ e_2^2)$.
b. (1 point) The first-order conditions with respect to the two types of efforts are $b_1-e_1=0$ and $b_2-e_2=0$. Therefore, both types of effort are equal to the marginal payment for each type of output.
c. (1 point) $E[q-w]=e_1+e_2-E[w]$. Now, the participation constraint gives us that $E[w]=RP^A+c(e_1,e_2)$, or $E[w]= 0.5b_1^2+0.5b_2^2+0.5b_1b_2+0.5(e_1^2+e_2^2)$. Therefore, incorporating also the ICC, we have that $E[q-w]= b_1+b_2- 0.5b_1^2 - 0.5b_2^2 -0.5b_1b_2 -0.5(b_1^2+b_2^2)$, or simplifying $b_1+b_2- b_1^2 - b_2^2 -0.5b_1b_2$.
d. (1 point) The first-order conditions for b_1 and b_2 are $1-2b_1-0.5b_2=0$ and $1-2b_2-0.5b_1=0$. Solving these two equations yields $b_1=b_2=2/5$
e. (1 point) $E[q]=e_1+e_2=b_1+b_2=4/5$.

3. A hospital manager has to design a contract for a new pharmacist. The patient satisfaction (q) depends on the pharmacist's actions according to q=e+u, where u is a random variable with a mean of zero and a variance of 1. The pharmacist's cost of action function is $0.5e^2$. The manager cannot observe the pharmacist's effort. If the pharmacist is trustworthy, when she signs the contract she will take the efficient action, even if this action cannot be verified. On the other hand, if the pharmacist is not trustworthy, she will take only actions that can be verified. The manager knows that λ of pharmacists in the market are trustworthy. Assume that the manager and all pharmacists in the market are risk neutral and have an outside option of 0. The contract is of the form (w,e), where w is a fixed payment and e is a desired action.

   a. (1 point) Suppose the manager specifies in the contract the efficient level of effort. What level of effort will then the trustworthy and untrustworthy pharmacists supply?
   b. (1 point) If the manager wants to make the contract acceptable to the trustworthy pharmacist, what fixed payment will she offer?
   c. (1 point) What is the expected utility of the trustworthy and untrustworthy pharmacists, given that the fixed payment is such that it is acceptable to the trustworthy agent?
   d. (2 points) Show that the manager will find it optimal to hire a pharmacist only if $\lambda > 0.5$.

Answer

   a. (1 point) The efficient level of effort is that which equates the marginal benefit and the marginal cost, or e*=1. By assumption, the trustworthy pharmacist will then take e*=1 and the untrustworthy pharmacist will take e=0.
   b. (1 point) $E[U]=w-c(e)=w-0.5e^2=R=0$. For the trustworthy agent, e=1, therefore w=0.5.
   c. (1 point) $E[U]=0.5-0.5(1)^2=0$ for the trustworthy pharmacist and $E[U]=0.5-0.5(0)^2=0.5$ for the untrustworthy pharmacist.
   d. (2 points) The expected profit for the manager is $E[V]= \lambda(1)+(1-\lambda)(0)-0.5=\lambda-0.5$. This is greater than the manager's outside option of 0 if and only if $\lambda > 0.5$.

4. Consider a promotion between a man and a woman. The productivity of contestants is given by $q_M = e_M + 0.5u$ for the man and $q_w = e_w - 0.5u$ for the woman, where e is the effort level and u is a random variable that is distributed uniformly on [-1,1]. The cost of effort is $0.5e_M^2$ for the man and e_W^2 for the contestants. Both contestants are risk neutral and both have an outside option of R=0. The affirmative action law stipulates that the man wins only if $q_M > q_W + 1$.

   a. (1 point) What is the efficient level of effort for each man and woman?
   b. (2 points) Using the incentive compatibility constraint for the man, what is the optimal prize spread between the winning and losing wage (W-w) that induces the man to supply the efficient level of effort?
   c. (1 point) If the contract induces the efficient level of effort from both contestants, what is the probability that the man wins?
   d. (1 point) What is the minimum losing wage (w) that will induce the man to participate in the contest?

Answer

a. (1 point) The efficient level of effort for each contestant equates the marginal benefit and cost of effort. Therefore, we have that the efficient level of effort for the man is $e_M=1$ and for the woman it is $e_W=0.5$.
b. (2 points) For the man, $E[U]=pW+(1-p)w-0.5e_M^2$. Now, $p=Prob(q_M>q_w+1)=Prob(e_M+0.5u> e_w-0.5u+1)$, or $Prob(u>1+e_w-e_M)$. Given that u is distributed uniformly on [-1,1], this simplifies to $(1-1-e_W+e_M)0.5=0.5(e_M-e_W)$. Therefore, $E[U]=w+(W-w)0.5(e_M-e_W)- 0.5e_M^2$. The first-order condition is then $0.5(W-w)-e_M=0$. Now, $e^*=1$ for the man, from which it follows that W-w=2.
c. (1 point) $p=0.5(e_M-e_W)=0.5(1-0.5)=0.25$.
d. (1 point) $w+p(W-w)-0.5e_M^2=w+0.25(2)-0.5(1)^2=w+0.5-0.5\geq 0$, so $w\geq 0$.

5. At research-based universities, professors who do not publish research articles in academic journals are typically fired, while at teaching-based universities, professors stay employed even if they do not publish research articles. To investigate whether the research output is higher in the research-based universities as compared to the teaching-based universities, a consulting firm has collected data and estimated the following model: $y_i=a+bD_i+u_i$, where i indexes professors, y represents the research output, D=1 for professors employed at the research universities and 0 for professors employed at the teaching universities, u is the impact of all other factors on research output (with a mean of zero), and a and b are coefficients to be estimated.

   a. (1 point) Interpret the meaning of coefficient *a* in this model.
   b. (1 point) Interpret the meaning of coefficient *b* in this model. Do you expect this coefficient to be positive/negative/zero? Why?
   c. (2 points) Discuss why comparing the research output of professors in the two types of universities will not in general identify the impact of compensation method. Explain how a randomized experiment can solve this problem.
   d. (1 point) Discuss two main shortcomings of using randomized experiments to discover the incentive impact of various compensation contracts.

Answer

a. (1 point) a represents the average research output of professors employed by teaching universities.
b. (1 point) b represents the incremental average research output of professors at research universities relative to professors at teaching university. We expect b to be positive because their pay is related to their research output.
c. (2 points) The two groups of professors may be different in other ways than the fact that they are employed by the two types of universities, such as their ability. A randomized experiment can solve this problem by randomly assigning professors to the type of university.
d. (1 point) Randomized experiments are in general quite costly or difficult to implement for ethical reasons, and the results may not easily generalize.

6. The shareholders consider designing a contract for a prospective portfolio manager. The value of portfolio is given by q=e+u, where e is the manager's effort and u is a random variable with a mean of zero and a variance of one. The manager is risk-averse, with the coefficient of absolute risk aversion equal to one, while the shareholder are risk neutral. The outside options are zero for both the manager and the shareholder. The compensation consists of a base salary plus a percentage of the portfolio value.

   a. (2 points) What are the manager's and shareholder's expected payoffs when the manager's effort can be observed by the shareholders?
   b. (3 points) What are the manager's and shareholder's expected payoffs when the manager's effort cannot be observed by the shareholders?

Answer

a. If the effort can be observed, the shareholders should completely insure the manager since they are risk neutral while the manager is risk averse. This can be achieved by paying a fixed salary in exchange for the optimal level of effort that satisfies the manager's participation constraint. E[U]=a-c(e)=R, which in this case translates to $a=0.5e^2$. The shareholders then choose the effort level e to maximize $E[V]=E[q]-a=e-0.5e^2$, which implies the optimal level of effort of e*=1 and E[V]=1-0.5=0.5. On the other hand, the manager's expected payoff is just his outside option, which in this case is equal to 0.
b. If the effort cannot be observed, the contract must balance incentives and insurance objectives. With a linear contract of the form a+bq, the manager's expected payoff is $E[U]=E[w]-c(e)-RP=a+be-0.5e^2-0.5(1)b^2(1)$. The incentive compatibility constraint then yields $\partial E[U]/\partial e=0=b-e$, or e=b. The shareholders' expected payoff then becomes E[V]=E[q]-E[w]. The participation constraint for the manager yields $E[w]=c(e)+RP=0.5e^2+0.5b^2$. Then, $E[V]=e-0.5e^2+0.5b^2$. With the ICC (e=b), this becomes $E[V]=b-b^2$. The first-order condition for b is then 1-2b=0, which yields b=0.5. The shareholder's payoff is then $b-b^2=0.25$. The manager's payoff is still equal to his outside option of 0.

7. Based on your course readings, discuss each of the following statements. For each statement, use one reading only and refer to the reading by the specific authors' names. (Recall that the required readings included: Shearer, Gibbons and Murphy, Hannaway, Kerr, Knez and Simester, Becker and Huselid).

   a. (1 point) Piece rate contracts are less effective than hourly wage contracts in motivating employees to work hard.
   b. (1 point) CEOs' pay is in general positively related to performance of other firms.
   c. (1 point) It is sometimes optimal to redesign an existing job into two jobs, each with distinct tasks.
   d. (1 point) It is sometimes better not to link pay to performance, even if measures of performance are readily available.
   e. (1 point) Team compensation is unlikely to improve employees' performance due to the free rider problem.

Answer

a. (1 point) Shearer shows that the opposite is true using a randomized experiment for tree planters in B.C. Specifically, he shows that the piece rate planters are about 20 percent more productive than planters paid based on hourly wage.
b. (1 point) Gibson and Murphy show that the compensation in CEOs is negatively related to performance of other firms. The use of relative compensation typically means that the uncertainty in pay can be reduced and incentive improved.
c. (1 point) Hannaway shows that this can work for teachers, when the school board cares both about simple teaching tasks that can be accurately measured and complex teaching tasks that cannot be accurately measured. In this case, combining both tasks in one job introduces uncertainty that lowers incentives. A better solution is to separate the two tasks, and strongly link pay to performance in the job in which performance can be easily measured.
d. (1 point) Kerr discusses examples from politics, sports, etc. where readily available performance measures can be dysfunctional, and the principal would be better off by not linking pay to performance.
e. (1 point) The negative impact of the free rider problem can sometimes be overcome by the peer pressure. Knez and Simester document this in their study of Continental Airlines in which monthly group bonus improved the on-line performance.

THE END